AD-A232 192

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. Agency Use Only *(Leave blank)*.

2. Report Date.
1990

3. Report Type and Dates Covered.
Proceedings

4. Title and Subtitle.
Scattering from Rigid and Fluid-Loaded Elastic Objects

5. Funding Numbers.
Program Element No 61153N
Project No 03208
Task No 010
Accession No DN255011

6. Author(s).
M. F. Werby

7. Performing Organization Name(s) and Address(es).
Naval Oceanographic and Atmospheric Research Laboratory
Ocean Sciences Directorate
Stennis Space Center, MS 39529-5004

8. Performing Organization Report Number.
PR 90:020:221

9. Sponsoring/Monitoring Agency Name(s) and Address(es).
Naval Oceanographic and Atmospheric Research Laboratory
Ocean Sciences Directorate
Stennis Space Center, MS 39529-5004

10. Sponsoring/Monitoring Agency Report Number.
PR 90:020:221

11. Supplementary Notes.
ICRD

12a. Distribution/Availability Statement.
Approved for public release; distribution is unlimited.

12b. Distribution Code.

13. Abstract *(Maximum 200 words)*.
The computation of scattered fields from fluid-loaded bounded objects can be treated in a consistent manner using the extended boundary condition (EBC) or T-matrix method for electromagnetic, elastic, and acoustic problems. We outline the general principles of the EBC method and apply it to problems of engineering interest which pertain to acoustical scattering from submerged three-dimensional elongated impenetrable objects or elastic solids and shells. Several physical examples of interest are offered and interpreted.

14. Subject Terms.
(U) Acoustic Scattering; (U) Shallow Water; (U) Waveguide Propagation

15. Number of Pages.
8

16. Price Code.

17. Security Classification of Report.
Unclassified

18. Security Classification of This Page.
Unclassified

19. Security Classification of Abstract.
Unclassified

20. Limitation of Abstract.
SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

91 2 11 182

PROCEEDINGS

INTERNATIONAL CONGRESS ON RECENT DEVELOPMENTS IN AIR- AND STRUCTURE-BORNE SOUND AND VIBRATION

MARCH 6-8, 1990 AUBURN UNIVERSITY, USA

Edited by
Malcolm J. Crocker

Volume 2

INTERNATIONAL CONGRESS ON
RECENT DEVELOPMENTS IN AIR- AND
STRUCTURE-BORNE SOUND AND VIBRATION
MARCH 6-8, 1990 AUBURN UNIVERSITY, USA

SCATTERING FROM RIGID AND FLUID-LOADED ELASTIC OBJECTS

M. F. Werby
Code 221, Numerical Modeling Division
Naval Oceanographic and Atmospheric Research Laboratory
Stennis Space Center, MS 39529
U.S.A.

ABSTRACT

The computation of scattered fields from fluid-loaded bounded objects can be treated in a consistent manner using the extended boundary condition (EBC) or T-matrix method for electromagnetic, elastic, and acoustic problems. We outline the general principles of the EBC method and apply it to problems of engineering interest which pertain to acoustical scattering from submerged three-dimensional elongated impenetrable objects or elastic solids and shells. Several physical examples of interest are offered and interpreted.

INTRODUCTION

Several techniques are available for describing waves that scatter from objects of known constitution and geometry. Many of them are rather specific or have intractable numerical pitfalls. It is therefore desirable to obtain a formulation that allows for general objects, frequency ranges, and boundary conditions, and that overcomes numerical difficulties commonly encountered in several broad classes of numerical methods. In this paper we briefly describe a consistent, unified and manageable numerical approach useful for researchers interested in solving any of a wide class of scattering problems. It is based on the coupling of the exterior and interior solutions of the surface boundary representation of the Helmholtz or elastodynamic equations and yields the Extended Boundary Condition (EBC) method of Waterman[1,2]. The EBC method avoids numerical problems often encountered by other techniques. We present numerous physical examples which are chosen not only for their intrinsic interest but also because they represent comparatively difficult problems to solve by other means.

EBC FORMULATION

The EBC technique is a boundary integral method that couples the exterior direct scattering field solution with the interior field solution. The method was initially developed for electromagnetic scattering by Waterman[1] in 1965, and subsequently for acoustical scattering by him[2] in 1969. The method has since evolved and is employed in all areas of classic scattering at various levels of sophistication[3-4]. As with other methods, it is often possible to make both structural and numerical variations in the basic EBC approach. We have developed formulations[3-5] which in our experience have proven the most favorable for numerical implementation and we employ them in this work. We begin with the simplest formulation, for scattering from an impenetrable object of rotation.

The Huygen's-Poincare' integral representation of the scattered field U exterior to the bounded object, can be described by the following equation:

$$U(r) = U_i(r) + \int_s [U_+(r')\partial G(r,r')/\partial n - G(r,r')\partial U_+(r')/\partial n]dS. \quad (1)$$

The quantity, U_+ is the scalar wavefield on the object surface, G is the outgoing Green's function, r' is taken on the surface of the object, and n is a unit outward normal vector to the surface of the object. S is taken to be the surface of the bounded object. In the subsequent development for elastic targets, U_+ will be the displacement vector, and G the Green's dyadic. An additional expression required can be obtained by expressing the Huygen's-Poincare' relation at a point interior to the object, which leads to the following equation:

$$0 = U_i(r'') + \int_s [U_+(r')\partial G(r'',r')/\partial n - G(r'',r')\partial U_+(r')/\partial n]dS \quad (2)$$

where r" is an object interior point. In this case, the field can be described as being "nulled"; hence, the nomenclature "null field" method.

Eqs. (1) and (2) yield the extended boundary condition equations. Since in their present form they are not directly useful, we now reduce them to a form amenable to numerical computation. Elastic targets submerged in a fluid require far greater mathematical detail, as indicated by Pao[6] and the Varadans[7]. Let us assume that $\partial U_+(r')/\partial n = 0$, so that we obtain the expressions

$$U(r) = U_i(r) + \int_s [U_+(r')\partial G(r,r')/\partial n]dS \quad (3)$$

and

$$0 = U_i(r) + \int_s [U_+(r')\partial G(r,r')/\partial n]dS. \quad (4)$$

To solve these expressions, it is convenient to represent $U_i(r)$, $U_+(r')$ and $G(r,r')$ in some suitable series expansion, which upon truncation leads to matrix equations that can be solved using digital computers. The Green's function, G, is a normal operator, and thus can be represented by the biorthogonal series

$$G(r,r') = ik\Sigma \mathrm{Re}\,\varphi_i(r<)\varphi_i(r>) \quad (5)$$

where r< and r> are the greater and lesser of the two points r and r' relative to the origin of the object. U_i, the incident wavefield, is known and can be shown to be:

$$U_i(r) = \Sigma a_n \mathrm{Re}\varphi_n(r). \quad (6)$$

Similarly the scattered field $U_s(r) = U(r) - U_i(r)$ can be expanded as:

$$U_s(r) = \Sigma f_n \varphi_n(r) \quad (6')$$

which leads to

$$a_n = -ik\int U_+(r')\,\partial\varphi_i(r')/\partial n\, dS. \quad (7)$$

and

$$f_n = ik\int U_+(r')\,\partial \mathrm{Re}\varphi_i(r')/\partial n\, dS. \quad (7')$$

The following expansion has been proven to satisfy closure on the surface for the rigid problem:

$$U_+(r) = \Sigma_n b_n \mathrm{Re}\varphi_n(r) \quad (8)$$

where b_n is the only unknown on the right hand side of Eq. (7). This reduces (7) to an expression in which the expansion coefficients b_n are the only quantities to be determined. We obtain

$$a_n = -\Sigma_m b_m \int\varphi_m(r')\partial\, \mathrm{Re}\varphi_n(r')/\partial n\, dS = -\Sigma_m b_m Q_{mn}, \quad (9)$$

$$f_n = \Sigma_m b_m \int \mathrm{Re}\varphi_m(r')\partial\, \mathrm{Re}\varphi_n(r')/\partial n\, dS = \Sigma_m b_m \mathrm{Re}Q_{mn}, \quad (9')$$

where Q_{mn} is an element of some known matrix. In matrix form, Equations (9) and (9') can be expressed

$$a = -ikQb \tag{10}$$

$$f = ik \text{ Re } Qb. \tag{11}$$

There are a variety of numerical strategies to attack this computational problem. The most straightforward (but unfortunately the most problematic) method would be direct numerical inversion, to eliminate b from Eq. (10) and replace it in Eq. (11), to arrive at a formally convenient expression for the scattered wavefield, represented by f. This leads to Eq. (12), relating the scattered field f with the incident field a.

$$f = -\text{Re } QQ^{-1}a. \tag{12}$$

Q is an $n \times n$ matrix, and a and f are $1 \times n$ column vectors. We can view this as a mapping from the incident field a to the scattered field f via the quantity $-\text{Re}QQ^{-1}$. It is conventional in certain areas of physics to describe such a mapping as a transition; hence this computational mapping is often referred to as a transition or "T matrix" operation.

As noted, a straightforward evaluation of Eq. (12) is usually *not* the most efficient way to solve for f. An alternate method devised by us to obviate this difficulty is the coupled higher-order T-matrix[3,4] method described below. The most elementary formulation of this solution is to block Q into four submatrices (Q(ij), where ij = (l, m) and demand that the Q(ii) matrices be square. This results in a new T matrix of order (l, l) which is convergent and requires an inverse no larger than the largest pair of (ii).

$$T(ll) = -PG^{-1} \tag{13}$$

where

$$P = \text{Re } Q(ll) - \text{Re } Q(lm)\, Q^{-1}(mm)\, Q(ml) \tag{14}$$

and

$$G = Q(ll) - Q(lm) Q^{-1}(mm)\, Q(ml). \tag{15}$$

Eq. (10) is related to an integral equation of the first kind, and can be transformed using a method developed by us into an eigenvalue problem. The unknown quantity in Eq. (10) is the surface term associated with the column vector b. By obtaining b directly, we then have the solution for the scattered field f via Eq. (11). To accomplish this we create a Hermitian matrix $H = Q\,Q^{\dagger}$, where $Q^{\dagger}$ is the adjoint of Q. (It is easy to show that H is self-adjoint or Hermitian). The eigenvalues λ_i and the eigenvectors β_i can be obtained in a straightforward manner, described below, where

$$H\,\beta_i = \lambda_i\,\beta_i. \tag{16}$$

It is known that the eigenvalues λ_i of Hermitian operators are positive, real and monotonically increasing, and that the eigenvectors β_i form an orthonormal set of vectors. Moreover, it can be shown that these vectors span the space associated with this problem. Thus, we can expand the unknown surface quantity b in terms of the β_i's. By exploiting the orthonormality of the eigenfunctions of H, we arrive at the following expression for f:

$$f = -\text{Re } Q\,\beta\; 1/\lambda (Q\,\beta)^{\dagger}. \tag{17}$$

The more complicated case of elastic targets imbedded in a fluid may also be derived, in which many of the previous procedures can be used. Let us outline the solution of this problem for an elastic solid. We still require the Huygen's-Poincare' integral in the fluid due to scattering from the target. However, we also require a generalization of the expression for an object in an elastic environment. An expression suitable for the problem we wish to develop has been obtained by Varatharajulu and Pao[6] in a form appropriate to derive a transition matrix for elastic targets, and will be used in the following equations. For the fluid we have:

$$U(r') = U_i(r') + \int [t_+ U - t(U)\ \ U_+]\, dS \qquad \text{(interior point)} \tag{18}$$

$$0 = U_i(r'') + \int [t_+ U - t(U)\ \ U_+]\, dS \qquad \text{(exterior point).} \tag{19}$$

Here traction $t = n \cdot \{\lambda I\ \nabla U + \mu(\nabla U + U\nabla)\}$. In the solid, we have an analogous pair of equations

$$U(r'') = U_i(r'') + \int [t_- U - t(U)\ \ U_-]\, dS \qquad \text{(interior point)} \tag{20}$$

$$0 = U_i(r') + \int [t_- U - t(U)\ \ U_-]\, dS \qquad \text{(exterior point)} \tag{21}$$

where r" is in the object interior and r' the exterior. t_- (t_+) and U_- (U_+) are the traction and displacement vectors on the surface as approached from the object's interior (exterior).

Imposed on these equations we have the following boundary conditions:

$$U_+ \cdot n = U_- \cdot n \tag{22}$$

$$t_+ \cdot n = t_- \cdot n \tag{23}$$

$$t \times n = 0. \tag{24}$$

In the following, we consider the case of general three-dimensional solids, using a spherical coordinate representation. Here the outward wave solutions to the full elastic wave equation are

$$\psi 1_{\sigma ml} = (\kappa/k)^{3/2}\ 1/k\ \nabla\ (h_l(kr)\ Y_{\sigma ml}\ (\theta,\phi)) \tag{25}$$

$$\psi 2_{\sigma ml} = 1/(l(l+1))^{1/2}\ \nabla \times (r h_l(kr)\ Y_{\sigma ml}\ (\theta,\phi)) \tag{26}$$

$$\psi 3_{\sigma ml} = 1/k\ \nabla \times (\psi 2_{\sigma ml}) \tag{27}$$

where h_l are spherical Hankel functions of the first kind, and Y_l^m's are spherical harmonics (normalized to unity). The total wavenumbers of longitudinal and transverse waves are k and κ. Normalization is chosen so that each wavefunction is normalized to a unit flux for any closed surface that encloses the origin. For a nonviscous fluid, only k is needed, so that (κ/k) is replaced by unity in Eq. (26).

As in the previous T-matrix formulation for acoustic targets, the general technique of obtaining a transition-matrix for an elastic solid is to eliminate the unknown surface wavefields and tractions. This elimination is accomplished by judicious use of the boundary conditions, the partial wavefield expansions (Huygen's principle for waves interior to the scatterer). The original derivation of the following expression is given by Bostrom[6]. By suitable elimination, one can arrive at relation (28), between the incident field partial wave coefficients {a} and scattered field partial wave coefficients {f}:

$$f = -Q_R^R\ R^{-1}P(Q_R^* R^{-1}P)^{-1}a. \tag{28}$$

For completeness we list the results for an elastic shell[7]:

$$f = -Q_R^R\ M^{-1}\ P(Q_R^* M^{-1}P)^{-1}a \tag{29}$$

where $M = Q_*^* + RT_2 + iT_2$, and Q, P, and R are defined by the following surface integrals:

$$Q(^{*}_{R}{}^{*}_{R})ij = k_1^3/\rho_1\omega^2 \int \{\lambda_1 \nabla \cdot (Ou/Re)\psi_*^{(1)}\ [n \cdot (Ou/Re)\psi_{ij}^{(2)}] \\ - [n \cdot (Ou/Re)]\psi_*^{(1)}]n \cdot t_2\ (Ou/Re)\ \psi_{ij}^{(2)}\} \cdot dS \tag{30}$$

$$R(^{o}_{R})_{ij} = k_{2s}^3/\rho_2\omega^2 \int \{ \zeta_2 [(\mathrm{Ou/Re})\, \psi_{ij}^{(2)}] \cdot [(\mathrm{Ou/Re})\, \psi_{ij}^{(2)} \times n] - (n \cdot [(\mathrm{Ou/Re})\, \psi_{ij}^{(2)}](n \cdot \zeta_2)[(\mathrm{Ou/Re})\, \psi_{ij}^{(2)} \times n] \} \cdot dS \tag{31}$$

$$P_{ij} = k_{2s}^3/\rho_2\omega^2 \int [(n \cdot \zeta_2)\, (\mathrm{Re}\, \psi_{ij}^{(2)}) n \cdot \psi_{s}^{(1)}] \cdot dS \tag{32}$$

$$T_2 = -\mathrm{Re}Q_2(Q_2)^{-1} \tag{33}$$

T_2 corresponds to scattering from an evacuated inclusion in an elastic environment. Without going into detail regarding the solution of Eqs. (28) or (29) it is best to employ the unitary method[5].

In the preceding, we have outlined methods that include a broad spectrum of boundary conditions to describe scattering from axially symmetric targets. Implementing these expressions is not without pitfalls, and much of our past efforts has focused on overcoming them, with good success. These methods are suitably general and effective to address a large class of problems without encountering complications experienced by other numerical schemes.

Below we apply the above EBC/T-matrix formulations to a number of physically illustrative engineering problems, that underscore some of the more dramatic computational features.

APPLICATIONS OF THE T-METHOD TO ENGINEERING PROBLEMS

We first treat scattering from rigid impenetrable objects. The two simplest cases are for spheroids and for cylinders with hemispherical caps. In addition, we also consider the case of scattering from elastic solids ,where we specifically examine resonance phenomena.

T-Matrix Applications to Impenetrable Problems

There are two classes of targets for impenetrable problems, i.e., soft and hard scatterers. They do not support body resonances; therefore; we examine acoustic quantities appropriate for nonresonant targets such as angular distributions which are dependent on target geometry and can be useful to determine such features of target shape as symmetry or elongation. In particular, reflection, diffraction, and generalized Snell's law behavior can be observed as curved-surface analogs for the plane-layered case.

Bistatic angular distributions correspond to measurement of a scattered field at any point in space for some incident wave fixed relative to some source-object orientation. In Fig. 1. we examine a rigid spheroid of aspect (length-to-width) ratio of 16:1. Fig. 1 a-d represent scattering from the object along the axis of symmetry (end-on) (a), 30 (b), 60 (c) and 90 (d) degrees relative to the symmetry axis (broadside). The values of the incident wavefield frequency are expressed using the dimensionless quantity $kL/2$, where L is the object length and k the total wavenumber ($k=2\pi/\lambda$). The value of $kL/2$ in Fig. 2 is 200, which implies that the object is about 70 wavelengths long and thus in the intermediate-frequency region where neither low nor high frequency approximations apply. In all figures frequency is sufficiently high that wave diffraction effects are significant in the forward scattering direction. Perhaps the most interesting feature of the four plots (Fig. 1 (a-d)) corresponds to a reflection at the (fairly flat) side of the object for scattering angles of 30 and 60 degrees. This reflection can occur only for very elongated objects that approach flat surfaces, so that the reflected angle is almost the same as the incident angle (relative to a straight line through the axis of symmetry).

EBC Applications to Elastic Solids

We now examine a phenomenon specific to elastic objects with smooth boundary conditions surrounded by an acoustic fluid, namely, body resonances. The body resonances examined originate from the curved-surface equivalents of seismic interface waves of pseudo-Rayleigh or Scholte type, propagating circumferentially to form standing waves on a bounded object. If phase velocities are slowly-varying (as a function of frequency) at the object surface, resonances occur at discrete values of $kL/2$. These resonances manifest themselves in a prescribed manner (described below). For elongated elastic solids, three distinct resonance types occur. The first kind that we wish to illustrate has to do with bending modes or flexural resonances. For unsupported spheroids a plane incident wave at 45 degrees relative to the axis of symmetry can excite these modes illustrated in Fig. 2a and 2b for aspect ratios of

4 and 5 to 1. It can be shown[9] that the lowest mode corresponds to 2, and thereafter 3, 4, etc. The interesting thing about these resonances is they can be predicted by exact bar theories and coincide nicely with result here. Of particular interest is the effect that with increasing aspect ratio the onset of resonances occur at lower kL/2 values as opposed to Rayleigh resonances. The second kind (at lower frequencies) are due to leaky Rayleigh waves and have been shown to be related to both target geometry and material parameters (notably shear modulus and density). Resonances can in this case best be observed by examining the back-scattered echo amplitude and phase response plotted as a function of kL/2. We illustrate this for an aluminum spheroid of aspect ratio 10 to 1 at end on incidence. Here we see two resonances superimposed on the semiperiodic pattern due to Franz waves associated with rigid scattering. If we subtract rigid scattering (in partial wave space) from the elastic response then we are left with the resonance response illustrated in Fig. 4 which shows that for aluminum the first resonance is broad while the second diminishes, in contrast to WC targets which yield small narrow resonances at the lower value and a larger resonance for the next value. In addition to the above wave phenomena, it is also possible to excite "whispering gallery" resonances, which we mention next. Finally, we examine broadside resonances for a 4 to 1 steel spheroid. Here we can excite three phenomena. At the lowest value we can see a spike representing the lowest order Rayleigh resonance seen end on corresponding to a standing wave circumnavigating the largest meridian of the spheroid. We also see weak Franz waves similar those due to a cylinder and than we see the lowest order Rayleigh and Whispering Gallery resonances corresponding to circumferential waves around the smallest meridian.

ACKNOWLEDGMENTS

Submitted under NOARL Contribution Number PR 90:020:221; approved for public release; distribution is unlimited.

REFERENCES

1. P. C. Waterman, Matrix formulation of electromagnetic scattering, Proc. IEEE, 53(3), (1965) 802.

2. P. C.Waterman, New foundation of acoustic scattering, J. Acoust. Soc. Am., 45, (1969) 1417.

3. M. F. Werby, & S. Chin-Bing, Numerical techniques and their use in extension of T-matrix and null-field approaches to scattering, Int. J. Comp. Math. Appls., 11(7/8), (1985) 717.

4. M. F. Werby and G. J. Tango, Application of the extended boundary condition equations to scattering from fluid-loaded bounded objects, Engineering Analysis, 5(1), (1988) 12.

5. M. F. Werby, and L. R. Green, An extended unitary approach for acoustical scattering from elastic shells immersed in fluids, J. Acoust. Soc. Am., 74(2), (1983) 625.

6. Y.-H. Pao, and V. Varatharajulu, Huygen's principle, radiation condition, and integral formula for the scattering of elastic waves, J. Acoust. Soc. Am., 60(7), (1976) 1361.

7. Acoustic, Electromagnetic, and Elastic Wave Scattering-Focus on the T-Matrix Approach, V. K. and V. V. Varadan (eds.), New York: Pergamon Press.

8. A. Bostrom, Scattering of stationary acoustic waves by an elastic obstacle immersed in a fluid, J. Acoust. Soc. Am., 67(2), (1980) 390.

9. M. F. Werby and G. C. Gaunaurd, Flexural resonances in obliquely insonified solid elastic spheroids, J. Acoust. Soc. Am., 85(6), (1989) 2365.

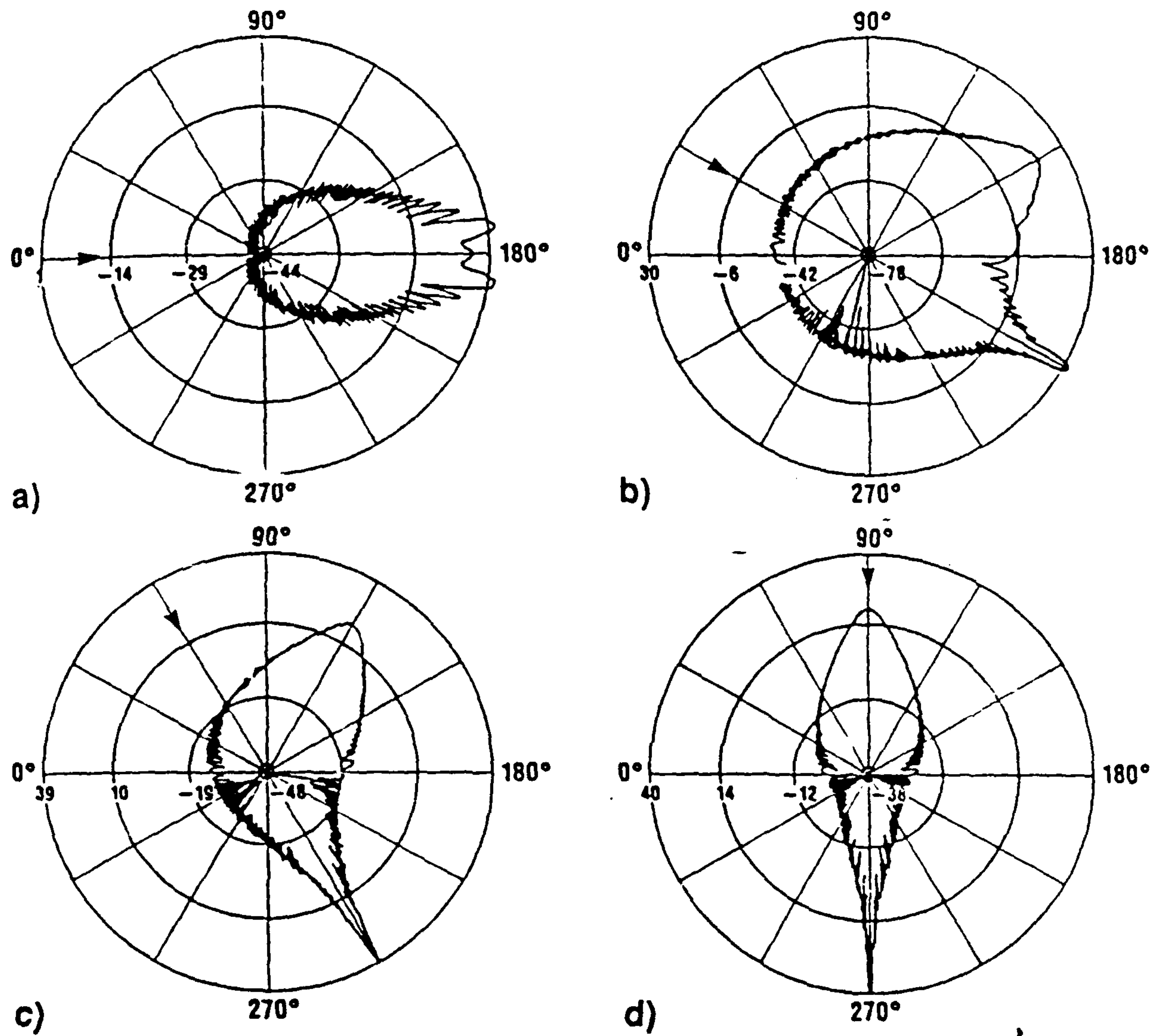

Figure 1. Scattering from rigid spheroid with aspect ratio of 16 to 1 at a kL/2 = 200 a) end on, b) 30 degrees relative to axis of symmetry, c) 60 degrees relative to axis of symmetry and d) broadside.

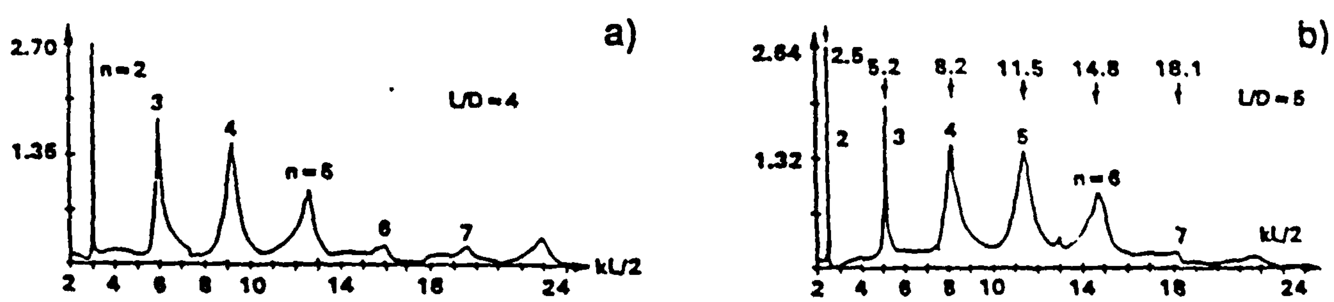

Figure 2. Flexural resonance predicted from a plane wave scattering at 45 degrees relative to the axis of symmetry of a steel spheroid a) of aspect ratio 4 and b) of aspect ratio 5.

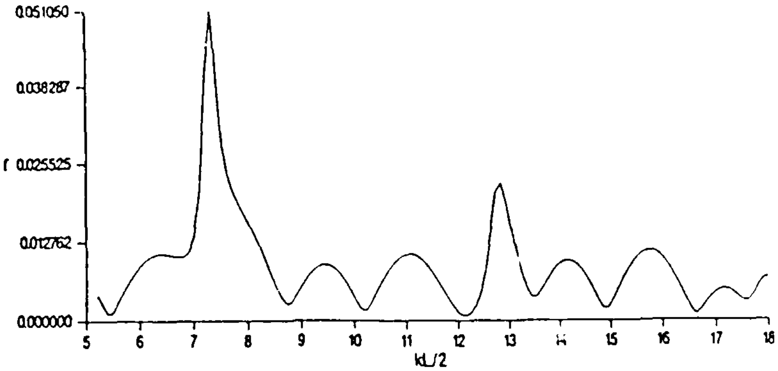

Figure 3. Back scattered signal from aluminum spheroid of aspect ratio of 10 to 1 end on incidence.

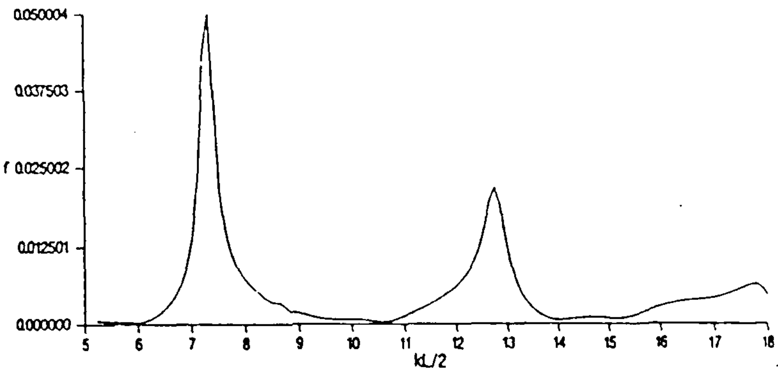

Figure 4. Back scattered signal from aluminum spheroid of aspect ratio of 10 to 1 end on incidence with rigid background subtracted.

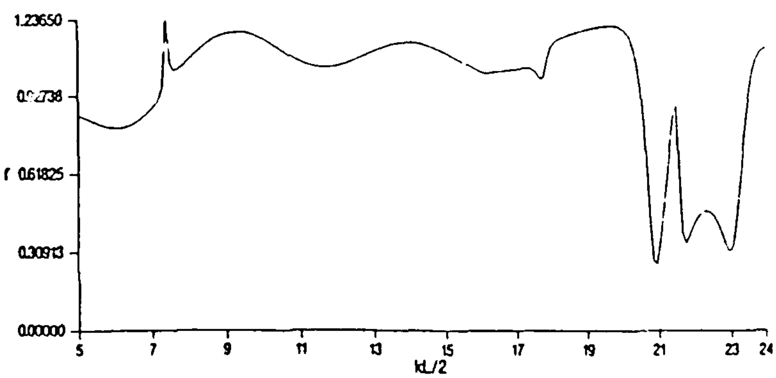

Figure 5. Back scattered signal from steel spheroid of aspect ratio of 4 to 1 broad side incidence.